Numerical Analyses of the Stress Distribution in Backfilled Stopes Considering Planar and Nonplanar Interfaces Between the Backfill and Rock Walls

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ABSTRACT

Minefill is widely used in underground mines around the world, mainly to improve ground stability, reduce ore dilution and maximize ore recovery rate. To successfully apply backfill, it is a critical issue to evaluate the stresses in backfilled stopes. Previous published works have shown that numerical modelling is an effective means to accomplish this task. However, most of the numerical models did not consider interfaces between the backfill and rock walls. The influence of the mechanical properties of the interfaces was seldom systematically investigated on the stresses in backfilled stopes. In this study, the stress distribution in backfilled stopes is analyzed using FLAC3D after taking into account interface elements. The influence of the planar and nonplanar interfaces, as well as the shear strength and roughness on the stresses in backfilled stopes will be presented and discussed.

INTRODUCTION

Minefill is increasingly utilized in underground mines due to its benefits for the mining operations, including a safer working platform or space, improved ground-stability conditions, increased ore-recovery rate and reduced ore dilution, also the reduced amount of mining wastes disposed on ground surface, which contributes to protection of the environment and geohazards reduction associated with the geotechnical instability of mining waste facilities (tailings dams or waste rock piles).

To successfully apply backfill in mine stopes, the interaction between the backfill and rock mass should be well understood, and the stress distribution in the backfilled stopes must be well estimated. This is particularly true to achieve good ground control, optimal backfill strength requirements design and safe mining operations with sill mats and barricades (Caceres 2005; Yumlu and Guresci 2007; Thompson et al. 2012; Sivakugan et al. 2013; Oulbacha 2014; Liu et al. 2016a,b,c).

When backfill is placed in mine stopes, it tends to settle due to its self-weight. But the stiffer rock walls tend to hold the backfill in place. This leads to the generation of shear stresses along the fill-wall contact interfaces and reduction of the vertical and horizontal stresses in the backfill. This phenomenon is known as “arching effect”. The stress reduction degree (i.e. degree of arching effect) depends on several influencing factors, including the conditions of contacts between the backfill and rock walls. In a stope with planar and smooth rock walls, the backfill can settle down freely, resulting in a minor degree of arching effect and a stress state close to that based on the overburden solution in the backfilled stope. In a stope with nonplanar and rough rock walls, the movement of the backfill would be difficult, resulting in significant arching effect and reduced stresses in the backfilled stope (Marston 1930; Li et al. 2003; Pirapakaran and Sivakugan 2007a, b; Liu et al. 2016 a, b).

Even though a number of analytical solutions have been developed for the stress evaluation under arching and can provide useful estimate of the stresses in backfilled stopes, especially in the preliminary stage of projects, they are usually developed for openings of simple and regular geometry with simplifying assumptions (Marston 1930; Aubertin et al. 2003; Li and Aubertin 2005; Pirapakaran and Sivakugan 2007a; Ting et al. 2011, 2014). If a more practical constitutive model for backfill and/or a more complex geometry for mine stopes must be taken into account, the numerical modellings will become necessary to obtain a more comprehensive understanding of the mechanical behavior of the backfill in interaction with the confining structures (rock walls and barricades).

To date, many works involving numerical modellings have been published for stress calculations in backfilled stopes (Caceres 2005; Li and Aubertin 2009; Veenstra et al. 2011a, 2011b; Veenstra 2013; Falaknaz et al. 2015). However, most of these numerical modellings did not consider interface elements between the backfill and rock walls (e.g., Aubertin et al. 2003; Li et al. 2003; Pirapakaran and Sivakugan 2007a, 2007b; Li and Aubertin 2009, 2015; Veenstra et al. 2011a, 2011b; Falaknaz et al. 2015). Only a few numerical analyses applied interface elements (Sivakugan et al. 2014; Ting et al. 2014), but these analyses were limited to interfaces with the same shear strength as the backfill.

In practice, due to the significant contrast in stiffness between the backfill and rock walls, the backfill particles can slip, rotate along and even separate from the rock wall surface. These fill-rock contacts are better to be treated as interfaces in the numerical modellings. In mining engineering, the rock walls of mine stopes are usually nonplanar due to production blasting and artificial supports. Smooth and planar interfaces can also occasionally be observed when the ore veins are in contact with discontinuities or joints. This indicates that the fill-wall interfaces can range from planar to nonplanar.

Up to now, the influence of the mechanical properties of these interfaces was seldom systematically investigated on the stresses in backfilled stopes, and the roughness of nonplanar interfaces in mine stopes has never been considered in previous numerical modellings for stress analysis.

In this study, the stress distributions along the vertical central line (VCL) of 2D typical backfilled stopes will be investigated by numerical modellings performed using FLAC3D (Itasca 2012) with the consideration of planar and nonplanar interfaces between the backfill and rock walls. The stresses obtained by numerical models with and without interface elements are compared. The asperities of nonplanar interfaces are idealized by saw teeth and the roughness of the interfaces is characterized by the saw teeth height and angle. The influence of filling layers, interface properties and geometry (saw teeth height and angle), and backfill properties on the stress distributions within backfilled stopes is also presented after considering planar and nonplanar interfaces.

NUMERICAL MODELS

To assess the stresses in backfilled stopes after accounting for interface elements, 2D plane-strain numerical models were created with FLAC3D, which is a 3D finite difference program using an explicit Lagrangian calculation scheme and a mixed discretization zoning technique (Itasca 2012). It has been shown to be a well-adapted tool for handling geomechanical problems, including mine-stope excavation and backfilling (Rankine 2004; Veenstra et al. 2011a, 2011b; Veenstra 2013; Liu et al. 2016a,b,c).

Fig. 1a shows a typical vertical backfilled stope with nonplanar interfaces between the backfill and rock mass. The asperities of the rock wall surfaces are idealized by saw teeth characterized by saw teeth height (h_t) and angle (θ). As a special case when $\theta = 180°$ ($h_t = 0$ m), the interfaces become planar. The stope is filled to a height of 40 m, with a void space of 0.5 m left between its roof and top surface of backfill. Its (nominal) width (*B*) is measured from the mid-height of the left saw teeth to the mid-height of the right saw teeth (*B* = 10 m for reference). By considering head-to-head or base-to-base width, one can define a minimum (B_{min}) or maximum (B_{max}) width. The rock mass is considered to be homogeneous, isotropic, and linearly elastic, characterized by γ_r = 27 kN/m^3 (unit weight), E_r = 30 GPa (Young's modulus) and μ_r = 0.25 (Poisson's ratio). The backfill is modelled as an elasto-plastic material obeying the Mohr-Coulomb criterion. It is characterized by γ = 18 kN/m^3 (unit weight), $E = 300$ MPa (Young's modulus), μ (Poisson's ratio), c (cohesion), ϕ (internal friction angle) and ψ = 0° (dilation angle). The interface elements are characterized by cohesion c_i, frictional angle δ, normal (k_n) and shear (k_s) stiffness.

Fig. 2b shows a typical 2D (plane-strain) numerical model constructed with FLAC3D with an enlarged view of the backfilled stope and interfaces. The bottom boundary of the numerical model is fixed in all directions, whereas the two lateral boundaries are only fixed in horizontal direction. All elements of the numerical model have a unit thickness in *Y*-direction, along which the displacement is prevented by using fixed condition. After a series of domain and mesh sensitivity analyses, the external boundaries are determined at 120 m from the stope walls and the optimal mesh size for the backfill is 0.2 m.

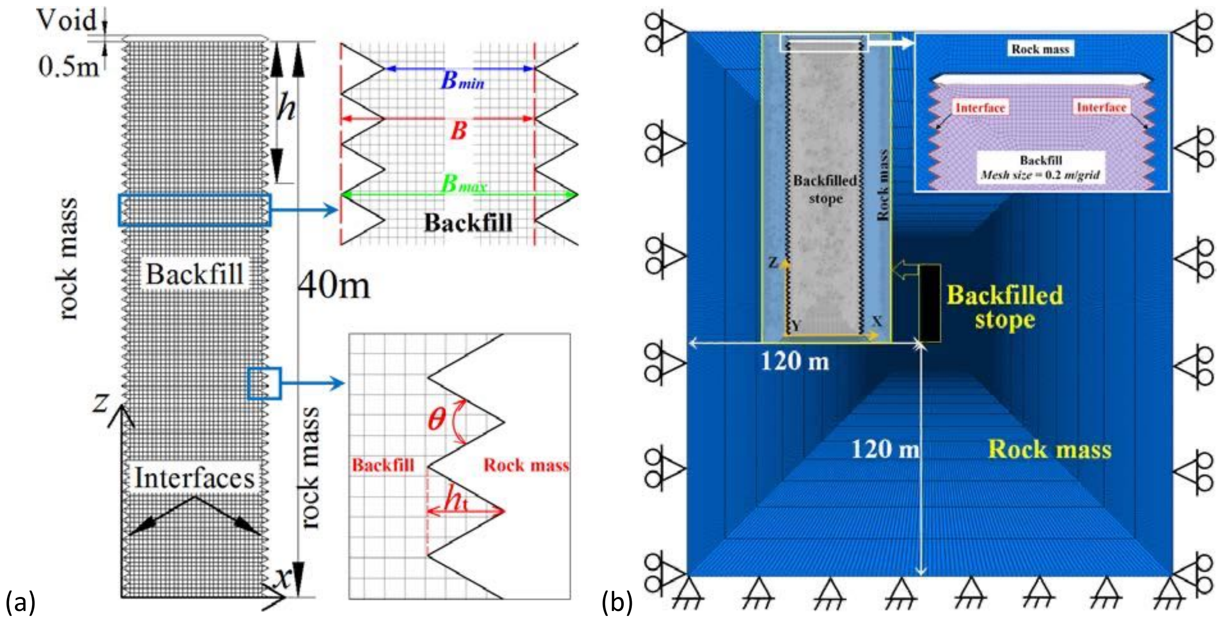

Figure 1. A typical vertical backfilled stope with nonplanar interfaces between the backfill and rock walls: (a) physical model; (b) numerical model constructed with FLAC3D

RESULTS ANALYSES

Before defining the program of numerical simulations, the stress variation in the backfilled stopes with the number of filling layers was first analyzed. The results (not shown here) indicated that the vertical and horizontal stresses become stable and insensitive to the further variation of the number of filling layers once this number reached 10. These results are somewhat different from those of Li and Aubertin (2009), who did not consider interface elements in their numerical models and reported that stable (static) results could be obtained using 4 layers of backfilling. This difference is the first one identified between numerical models with and without interface elements.

In the following simulations, the effect of several parameters on the stress distributions along the VCL of the backfilled stopes will be examined after considering interface elements. All numerical modeling steps were performed with 10 layers of backfilling (i.e., 4 m/layer).

Planar Interfaces ($\theta = 180°$, $h_t = 0$ m)

Fig. 2a presents the variation of the vertical (σ_v) and horizontal (σ_h) normal stress distributions along the VCL of the stopes obtained by 2D plane-strain numerical models with FLAC3D while varying the friction angle of interfaces, δ, from 0.4 to 1 times the friction angle of the backfill, ϕ. The stress distributions based on the overburden solution [$\sigma_v = \gamma h$; $\sigma_h = (1 - \sin\phi)\sigma_v$, where h is the depth, implying the application of Jaky's at-rest earth pressure coefficient] and those obtained by numerical modeling using reference parameters of the backfill (ϕ = 35° and c = 0 kPa) without considering interface elements are also plotted in the figure. Clearly, all vertical and horizontal stress distributions obtained by the numerical models are below the lines of the vertical and horizontal stresses based on the overburden solution. Thus, the arching effect occurs in the backfilled stope. Both the vertical and horizontal stresses decrease as the interface friction angle, δ, increases. An increase in the interface friction angle leads to a stronger arching effect along the rock walls and reduced stresses in the backfill. When the interface friction angle decreases, the arching effect diminishes, and the stress distributions obtained by the numerical models tend to approach those based on the overburden solution.

In Fig. 2a, it is interesting to note that the vertical and horizontal stresses along the VCL of the stope obtained by the numerical models without considering interface elements are smaller than those obtained by the numerical models considering interface elements as long as the interface friction angle (δ) is smaller than the backfill friction angle (ϕ). These results indicate that the application of interface elements is necessary to estimate the stresses in backfilled stopes as long as the rock-wall surfaces are smooth and planar because the interface friction is usually smaller than the backfill's friction angle. This is particularly the case in civil engineering, where the retaining wall or barricade structures are usually smooth and planar, and the interface friction angle is commonly near two thirds of the backfill's friction angle (Das 2004; CGS 2006). However, when the interface friction angle (δ) is equal to the backfill friction (ϕ), the stress distributions obtained by the numerical models with and without interface elements become nearly identical, indicating that the consideration of interface elements is not necessary in these numerical models. This is likely the case in mining engineering, where rock-wall surfaces are usually rough, and the interface friction angle is typically equal to the backfill's friction angle (Aubertin et al. 2003; Singh et al. 2010). But whether the contact interfaces between backfill and rock walls in mine stopes are always rough enough to realize the equal friction angle of interfaces and backfill are still unknown. It should be further investigated combined with interface shear tests and rock wall roughness quantification.

Fig. 2b presents the variation of the vertical and horizontal normal stress distributions along the VCL of the stope obtained by 2D plane-strain numerical modeling with FLAC3D before and after introducing interface elements when the backfill friction angle ϕ varies between 21° and 45°. The numerical models without interface elements were applied with c = 0. Before the interface elements are introduced, the vertical and horizontal stresses decrease as the backfill friction angle (ϕ) increases. These results are straightforward. An increase in the backfill friction angle increases the arching effect, thereby reducing the stresses in the backfilled stopes. However, it is surprisingly to note that, after introducing interface elements, both the vertical and horizontal stresses become almost insensitive to the variation of backfill friction angle ϕ from 21° to 45°. These results are considerably different from all the previous reports obtained by the numerical models before interface elements are introduced. The reason is that the numerical results presented here were obtained by considering interface elements with a constant friction angle of $\delta = 21°$. These results, along with those presented in Fig. 2a, tend to indicate that the stress state along the VCL of the stopes is controlled by the interface friction angle (δ), rather than the backfill friction angle (ϕ).

In practice, because the interface friction angle is usually proportional to the backfill friction angle, an increase in the backfill friction angle increases the interface friction angle and decreases the horizontal and vertical stresses. However, the shear strength parameters of interfaces are not always proportional to those of the backfill in mining engineering. For example, the shear strength parameters along an interface between a backfill and a rough and unweathered rock can be much higher than those between the same backfill and a smooth, planar and heavily weathered rock.

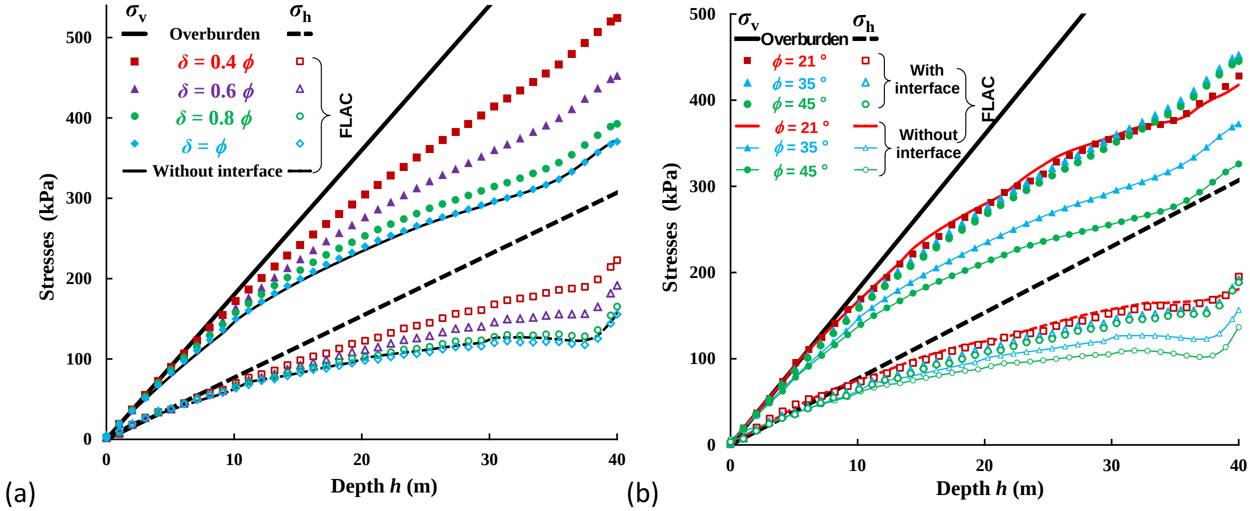

Figure 2. Vertical (σ_v) and horizontal (σ_h) stresses along the VCL of stopes with different (a) interface friction angle δ; (b) backfill friction angle ϕ obtained by numerical models with and without planar interface elements

Fig. 3a presents the variation of the vertical and horizontal normal stress distributions along the VCL of the stope obtained by 2D plane-strain numerical models with FLAC3D as a function of interface cohesion c_i. The stress distributions based on the overburden solution and those obtained by numerical models using the reference parameters of the backfill ($\phi = 35°$ and $c = 100$ kPa) without considering interface elements are also plotted in the figure. It can be seen that both the vertical and horizontal stresses decrease significantly when the interface cohesion increases from 0 kPa to 25 kPa. Once the interface cohesion exceeds 25 kPa, the vertical stress diminishes only slightly, whereas the horizontal stress becomes entirely insensitive to further increases in this parameter. A critical interface cohesion (approximately 25 kPa) close to that (approximately 30 kPa) reported by Li and Aubertin (2009), beyond which the stresses become insensitive to further increases in the cohesion, is also observed here. However, the vertical stress obtained without considering the interface elements is much smaller than that obtained with consideration of the interface elements. This is because the fill-wall contact strength is controlled by the backfill friction angle ($\phi = 35°$) in the former case without interface elements, but by the interface friction angle ($\delta = 21°$) in the latter case. This difference results in a more pronounced arching effect and smaller stress in the former case than in the latter case.

Fig. 3b shows the variation of the vertical and horizontal normal stresses along the VCL of the stope obtained by 2D plane-strain numerical models with FLAC3D before and after interface elements are introduced as the backfill cohesion *c* changes from 0 to 100 kPa. These results are obtained with a constant interface cohesion c_i = 0 kPa, and the numerical models without interface elements were applied with ϕ = 35°. Again, the variation of the vertical and horizontal stresses is relatively insensitive to the backfill cohesion *c* when it increases from 0 kPa to 100 kPa. These results, along with those shown in Fig. 2, also confirm that the stress state along the VCL of the stope is primarily controlled by the interface shear strength and is less controlled by the backfill's shear strength.

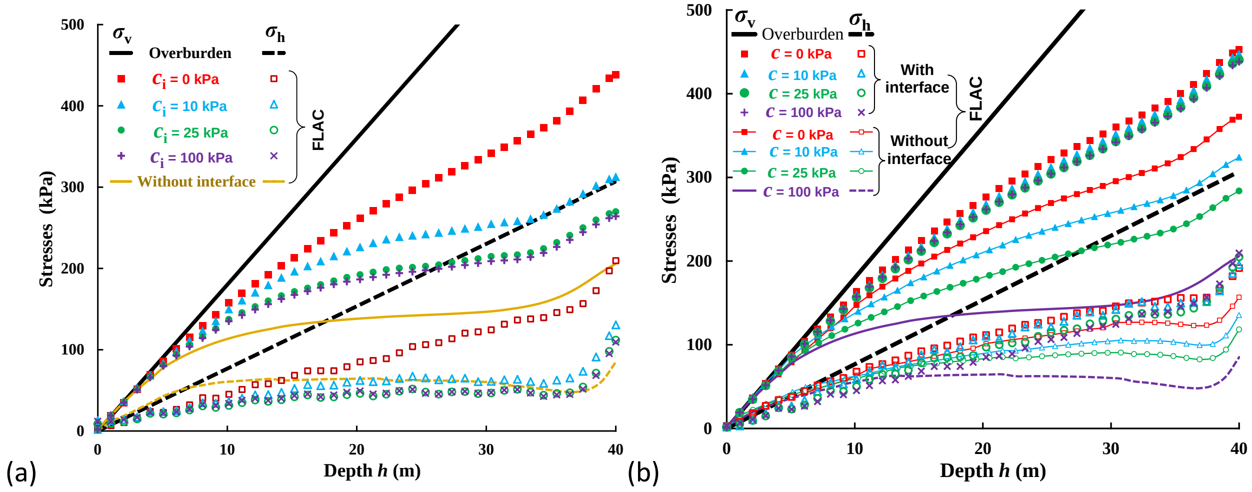

Figure 3. Vertical (σ_v) and horizontal (σ_h) stresses along the VCL of stopes with different (a) interface cohesion c_i; (b) backfill cohesion *c* obtained by numerical models with and without planar interface elements

Nonplanar Interfaces (0°<θ<180°, h_t > 0 m)

Fig. 4a shows the vertical and horizontal stresses distributions along the VCL of the stope with interfaces having different saw teeth heights, h_t. It can be seen that the stresses decrease significantly when the interfaces pass from planar (h_t = 0 m) to nonplanar (h_t = 0.2 m). With further increase in the saw teeth height from 0.2 m to 0.6 m, the vertical stress only deceases slightly whereas the horizontal stress keeps almost unchanged. These results clearly show that the roughness of the interfaces (i.e. rock walls of the stopes) should be taken into account in the stress estimation in backfilled stope.

Fig. 4b presents the vertical and horizontal stress distributions along the VCL of the stopes with interfaces having different saw teeth angles, θ. From the figure, one can see that the stresses generally decrease when the interfaces passes from a planar (θ = 180°) to a nonplanar ($\theta \neq 0$) state. However, the reduction in the vertical and horizontal stresses is quite different than that shown in Fig. 4a. This is because the results shown in Fig. 4a are obtained by fixing the saw teeth angle of the interfaces at θ = 120°. As long as the saw teeth have a certain height (i.e. ht > 0), the interfaces become rough. This results in a significant fall in both the vertical and horizontal stresses as shown in Fig. 4a.

Here, the saw teeth height of the interfaces is fixed at h_t = 0.3 m. When the teeth angle passes from θ = 180° to θ = 170°, the interfaces (rock walls) passes from planar to nonplanar. But the nonplanar interfaces are composed of a few segments only, resulting in limited roughness. Accordingly, the reduction in both the horizontal and vertical stresses is not as significant as that shown in Fig. 4a.

Fig. 4b also shows that the vertical and horizontal stresses can further decrease as the saw teeth angle θ further decreases from 170° to 120°. When the saw teeth angle θ goes from 120° to 60°, the vertical and horizontal stresses show a slight diminution only. These results are straightforward. With a decrease in the saw teeth angle, the saw teeth become sharper and the interfaces become rougher. Consequently, the shearing becomes more difficult along the fill-wall interfaces. This results in a more pronounced arching effect and decreased stresses in the backfilled stope. When the saw teeth angle θ further decreases from 60° to 30°, the vertical stress still decreases very slightly whereas the horizontal stress becomes completely insensitive to the variation of the saw teeth angle θ. This indicates that when the fill-wall interfaces are rough enough, the shearing mostly takes place within the backfill, rather along the very sinuous and rough planes of the fill-wall interfaces.

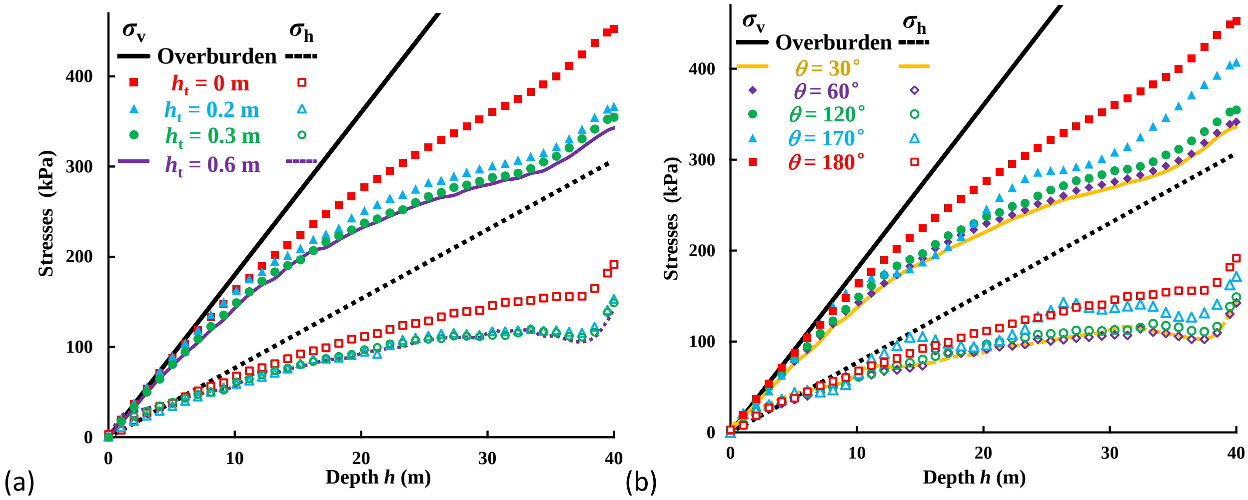

Figure 4. Vertical (σ_v) and horizontal (σ_h) stresses along the VCL of stopes with different rock wall roughness characterized by (a) sawteeth height h_t; (b) sawteeth angle θ obtained by numerical models with nonplanar interface elements

The results shown in Figs. 4 also indicate that the nonplanar interfaces defined in the reference case (θ = 120°, h_t = 0.3 m) are rough and the use of planar interfaces is not appropriate to present the nonplanar rock walls. But it is still unclear if these nonplanar interfaces are rough enough so that the numerical modellings can be performed without any consideration of fill-wall interface elements.

To clarify this issue, additional numerical modellings have been made with the same backfill properties and stope and fill-wall contact geometry, but without interface elements. The results are presented in Fig. 5. A part of the results obtained with nonplanar interfaces shown in Fig. 4b is also plotted on the Fig. 5. It can be seen that the nonplanar interfaces can only be considered rough enough when the saw teeth angle is as small as 30°, at which the numerical modellings with or without nonplanar interface elements result in the same stresses distributions. These results along with those presented in Liu et al. (2016a) partly justify the utilization of backfill's internal friction angle as that of the interfaces due to the irregularity of the rock walls in mine stopes. But it is clearly shown that the nonplanar interfaces in the reference case (θ = 120°, h_t = 0.3 m) are not sufficiently rough. Interface elements have to be used along these nonplanar fill-wall contacts to obtain a representative estimation of the stresses in the backfilled stopes.

Once the nonplanar interfaces are rough enough, the numerical modellings can be performed without considering interface elements. This is the conclusion drawn from the results shown in Fig. 5 for the case of θ= 30°. However, the roughness of the nonplanar interfaces here was simulated by regular saw teeth. For the practical rock walls in mine stopes, this is seldom the case. A more realistic way is to scan the surfaces of the rock walls in mine stopes by some specific apparatus (e.g., Cavity Monitoring Survey system) to obtain a more representative roughness condition of the rock walls.

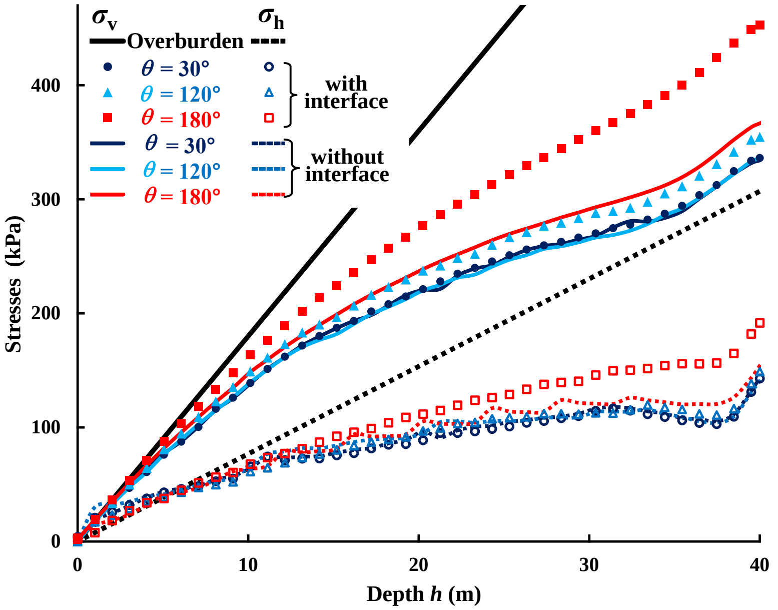

Figure 5. Vertical (σ_v) and horizontal (σ_h) stresses along the VCL of stopes with different rock wall roughness characterized by sawteeth angle θ obtained by numerical models with and without nonplanar interface elements

CONCLUSIONS

A series of 2D plane-strain numerical models were created with FLAC3D to evaluate the stress distribution along the VCL of backfilled stopes after accounting for the planar and nonplanar interfaces between the backfill and rock walls.

The results indicate that the vertical and horizontal normal stresses decrease as the interface shear strength increases. More specifically, the vertical and horizontal stresses decrease when the interface friction angle increases from 0.4 to 1 times the backfill friction angle. Given the stope geometry and material properties, the vertical and horizontal stresses decrease when the planar interface cohesion increases from 0 kPa to 25 kPa but become insensitive to further increases in the interface cohesion. The results also show that the vertical and horizontal stresses are rather insensitive to the variation in the shear strength of the backfill. This result is considerably different from those of most previous publications, which have indicated that the arching effect and the stress state are primarily controlled by the shear strength of the backfill.

It is interesting to note that it is necessary to consider the interface elements between the backfill and rock walls when the planar interfaces have a shear strength lower than that of the backfill. When the interface friction angle is equal to the backfill friction angle, the application of interface elements is not necessary in numerical models used to estimate the stresses in backfilled stopes.

In addition, the results show that the stresses in the backfilled stopes obtained by the numerical modellings with planar interface elements are much larger than those obtained by numerical modellings with nonplanar interface elements under the same conditions. This indicates that the use of planar interface elements in the numerical modellings tends to overestimate the stresses in backfilled stopes when the rock walls of mine stopes are nonplanar and rough. The results further show that the arching effect increases and the stresses in backfilled stopes decrease with an increase in the roughness of the interfaces. The roughness of the nonplanar interfaces can be increased either by an increase of the saw teeth height or/and a decrease in the saw teeth angle with the defined saw teeth interface model here. This indicates that it is important to take into account the actual geometry of the fill-wall interfaces.

Furthermore, it has been shown that it is not necessary to consider interface elements in numerical modellings if the interfaces are rough enough. For the stope and interface geometries and material parameters considered here, it has been shown that the interfaces become rough enough only when the saw teeth become very sharp, having a teeth angle of 30°. Once the interfaces are sufficiently rough, the numerical modellings can either be performed by considering the actual nonplanar fill-wall contacts without interface elements, or by considering planar fill-wall contacts without interface elements with the minimum (B_{min}) width.

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