Reliability Analysis of Mine Backfill Exposures

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ABSTRACT

The largest operating cost associated with most cemented mine backfill systems is binder. Binder dosage is typically based on the results of the quality control strength testing, a suitable model for estimating the strength requirements and a Factor of Safety (FoS). In the author's experience, safety factors (or equivalent) can range from 1.2 to 2.0, often with no rational basis for selection. At typical mine backfill operations this range may equate to a 1-3M USD difference in annual binder costs. Consequently, selection of a suitable FoS, which properly reflects the material and model uncertainty, is considered necessary.

This paper presents the results of analysis that was undertaken for the purpose of providing guidance relating to a suitable FoS for cemented mine backfill design. Work presented in this paper utilizes Monte Carlo simulations of a typical mine backfill vertical exposure to derive a rational approach for using variable quality control strength data (either plant sampling or in situ measurements) for the design of fill exposures.

In addition to providing a rational approach to addressing material uncertainty, this paper also provides recommendations relating to the selection of appropriate model uncertainty safety factors. While limited in scope the presented results provide useful guidance for the selection of appropriate model uncertainty safety factors when applying simplified analytical solutions, compared with more rigorous numerical methods.

INTRODUCTION

A standard component of most mine backfill operations is routine quality control (QC) testing. Even with strict procedures, QC strength test results are often quite variable. An example of this is presented in Figure 1, which shows the fluctuations in QC strengths for backfill batched with the same mix inputs (i.e. same binder and mix solids concentration). In addition, others (e.g. Le Roux et al. 2002) have identified similar variability in the strength of recovered core samples. Regardless of the method of quality control testing, the strength of cemented mine backfill has been shown to be variable and the nature of this variability is mostly random.

Conversion of this variable data into suitable cemented mine backfill mix design is not straight forward and how this is done could significantly impact the economics of a mine backfill operation through either excessive dilution or excessive binder consumption. This paper sets out to provide some guidance for addressing this.

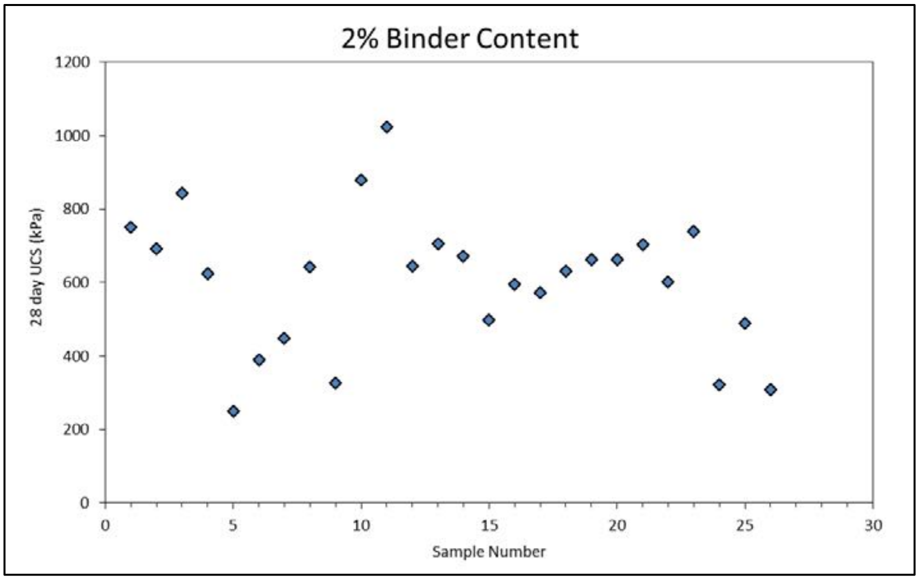


Figure 1. Variability in quality control sample strengths against chronological sample number

To investigate the impact of fill material variability on the stability of cemented mine backfill exposures, reliability analysis was undertaken. The objective of this analysis was to determine the material uncertainty, strength reduction factor ($\emptyset_{material}$). This strength reduction factor can be applied in a similar manner to that adopted in conventional design codes, where the capacity of the backfill material has to exceed the design strengths in accordance with:

$$Backfill\ Capacity\ \geq Design\ Strength$$

$$(\emptyset_{material})S \ \geq \ S^{*} = (\emptyset_{model})S \tag{1}$$

The primary focus of this paper is to derive a suitable strength reduction factor ($\emptyset_{material}$). However, in addition a limited amount of analysis was also undertaken to provide some guidance for the selection of a suitable model uncertainty load magnification factor ($\emptyset_{model}$).

It should be noted that these factors can be related to the FoS historically adopted in mine backfill designs through:

$$FoS = \frac{\emptyset_{model}}{\emptyset_{material}} \tag{2}$$

In addition, specification of the strength requirements for mine backfill exposures is most commonly carried out using either simplified analytical solutions or detailed numerical analysis. These two different approaches capture different levels of detail, specifically relating to the idealization of the material's constitutive behavior when defining the analytical solution. The second section of this document considers the sensitivity of the strength definition to this idealization and provides some useful outcomes relating to the load magnification factor that should be adopted to account for this.

MATERIAL UNCERTAINTY

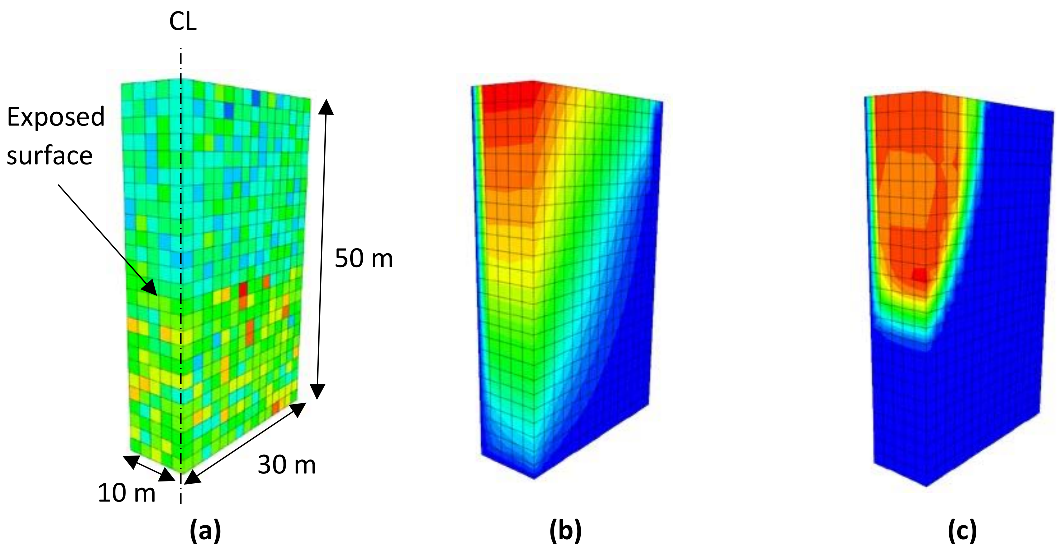
Reliability Analysis

In this paper reliability analysis is used to assess the effect of strength variability on the stability of an exposed fill mass. One particularly notable aspect of the observed QC and in situ fill strength variability is that in most cases, for a given mix, the strength varies randomly in accordance with a Gaussian distribution. This makes the scenario amenable to reliability analysis using Monte Carlo simulations. In this paper the effect of strength variability is assessed by undertaking Monte Carlo simulations where strengths are assigned randomly to various regions (or zones) of the model. This reliability analysis was undertaken using the finite difference software FLAC3D software (Itasca 2005). Analysis assumed a linear-elastic strain-softening constitutive model for the cemented mine backfill, with the assumption that:

- The elastic modulus is linearly related to the fill strength
- Beyond yield the bond strength strain-softens linearly with plastic shear strain

These assumptions are valid for almost all cemented mine backfill materials the authors have encountered. The adopted material relationships were based on the results of triaxial testing of a typical cemented mine backfill.

For this study, simulations considered a 20 m wide vertical exposure of a 50 m tall fill mass, where the fill mass was divided into two 25 m thick layers. Strengths were assigned randomly to individual zones (2 m × 2 m × 2m) based on a Gaussian distribution. The "half-space" of the model geometry is presented in Figure 2a, which shows the random distribution of zones of different strength. The displacement contour output of a typical stable and unstable model are presented in Figure 2b and c, respectively.

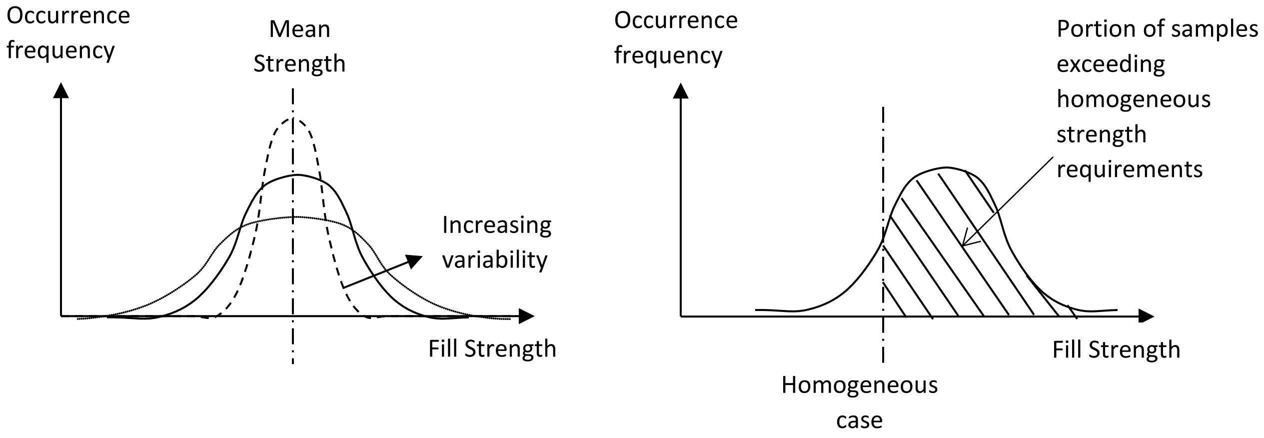


(a) (b) (c)

Figure 2. Model half-space for vertical exposure with (a) random zones (b) small stable displacements (c) large unstable displacements

Monte Carlo simulations were undertaken to determine the impact of strength variability relative to the homogeneous case. These simulations considered strengths assigned randomly in accordance with a range of different standard deviations. Standard deviations considered included 10%, 20%, 30%, 40% and 60% of the mean strength (i.e. coefficient of variation, CV). This range is representative of that expected to be encountered in a mine backfill situation.

The impact of increasing variability on the distribution of strength is illustrated in Figure 3a. To derive a framework for managing variable strengths, the Gaussian distribution curve was progressively shifted to the right (i.e. to increase strengths) and the portion of failure events determined for each. For each case, the position of the Gaussian distribution curve was determined based on the portion of samples exceeding the homogeneous strength requirement. This is illustrated in Figure 3b.

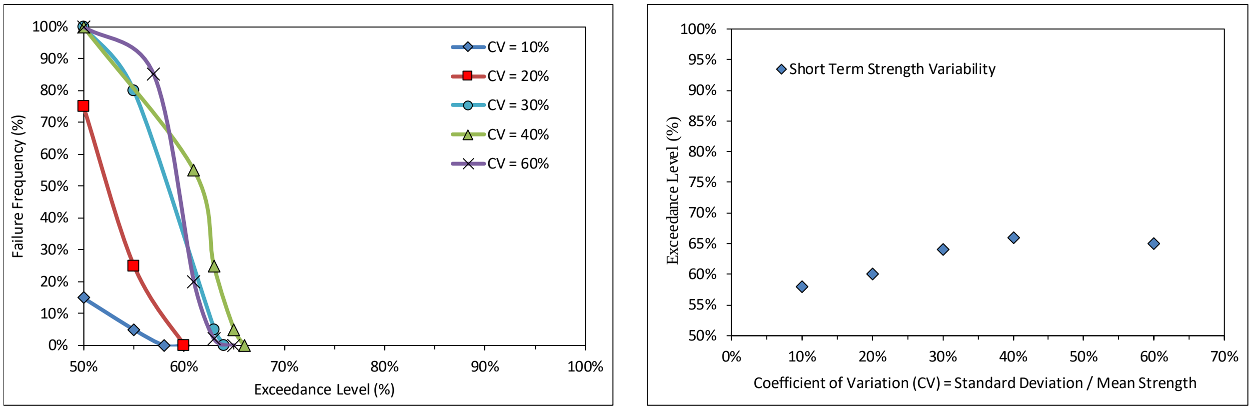


(a) **(b)**

Figure 3. Impact of (a) increasing standard deviations and (b) Exceedance level on the occurrence frequency for different fill strengths

For each CV and exceedance level combination a minimum of 20 vertical exposure Monte Carlo simulations were undertaken. The portion of models shown to be unstable is defined as the "failure frequency" for each case. The results of the Monte Carlo simulations are presented in Figure 4a. This figure shows the failure frequency plotted against the exceedance level for a range of different strength variations. Considering only the case of zero failure frequency (i.e. no failures) the portion of strength samples required to exceed the homogeneous failure strength for each CV was determined. This is plotted against CV in Figure 4b.

Figure 4b shows that, in order to avert fill mass failure the required exceedance level increases as the CV increases. However, where the CV is less than 60%, provided that 66% of the fill mass strengths exceed that required for stability under homogeneous conditions, the fill mass is expected to remain stable.



(a) (b)

Figure 4. (a) Failure frequency verses exceedance level for vertical exposures (b) Exceedance levels to maintain a zero failure frequency for different levels of variability

Significance of Zones Sizing

The selection of the zones sizes (or regions of constant strength) used in the reliability analysis has the potential to impact on the results of the Monte Carlo simulations. To investigate the significance of the selected zone size, a limited amount of analysis was undertaken with cubic model zones of dimensions 1 m, 2 m and 5 m. Using each of these model geometries analysis was undertaken to determine the relationship between exceedance level and failure frequency. This was undertaken for backfill of a uniform strength where the material strength varies in accordance with a CV of 30%.

The results of this analysis are presented in Figure 5. This figure shows that, compared with the 2 m and 5 m zones, where the zone size is reduced to 1 m a lower exceedance level is required to maintain stability. This is expected to be because as the zone size reduces, analysis trends towards the homogeneous case (where 50% exceedance would maintain stability). For the 2 m and 5 m zone sizes the required exceedance level is almost identical. This confirms that the (2 m) zone size adopted in the previous analysis is appropriate.

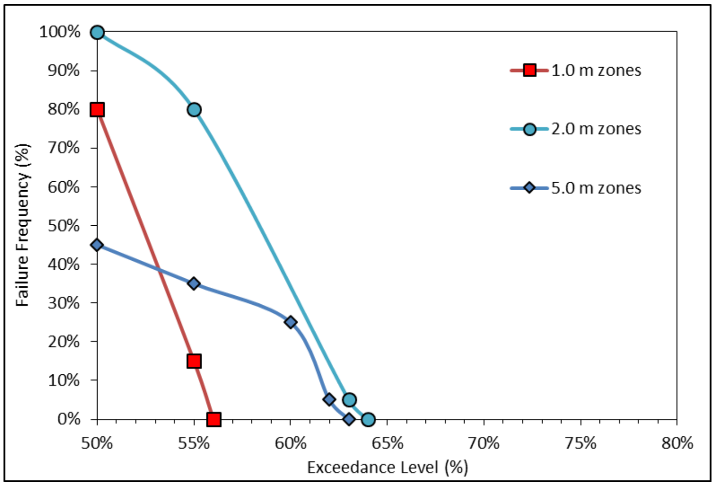


Figure 5. Comparison of zone sizing for Failure frequency verses Exceedance levels for different model zone sizes

Material Uncertainty - Framework

The results presented in the previous sections provide a rational philosophy for determining the Backfill Capacity ($\emptyset_{material}.S$) based on the results of variable quality control strength testing. This would be implemented using the following steps.

1. Confirm that the strength variability is random and the distribution is appropriately represented by a Gaussian distribution.
2. Determine the mean (μ) and the standard deviation (σ) of the measured quality control data for a particular backfill mix.
3. Based on the CV and acceptable failure frequency determine the required exceedance level from Figure 4a. Based on this exceedance level requirement determine the standard normal variate (*Z*) from standard Gaussian distribution charts (Montgomery and Runger, 2014).
4. Determine the appropriate capacity strength of the particular backfill mix according to:

$$Backfill\ Capacity\ (\emptyset_{material}.S) = \mu + Z\sigma \tag{3}$$

The results of the reliability analysis indicate that, provided the CV of the strengths is less than 60%, to ensure a failure frequency of zero it is necessary to have 66% of samples exceeding the required homogeneous strength. For an exceedance level of 66% the appropriate standard normal variant (*Z*) value is 0.413. The required strength reduction factor ($\emptyset_{material}$) can be determined in accordance with:

$$\emptyset_{material} = (1 - Z.CV) \tag{4}$$

In addition to the application of this framework for the conversion of operational quality control testwork data to mix designs, it is also useful to have an understanding of appropriate factors to apply to the results of laboratory testwork when estimating binder requirements during preliminary study phases of projects (i.e. prior to operation). Previous experience at many different cemented mine backfill operations indicate that QC strength testing typically results in a CV ranging from 15 to 30%. Using a CV of 30%, a 66% exceedance level target and the assumption that the laboratory results are representative of the mean strength (μ), in accordance with the logic described above, it would be appropriate to apply a strength reduction factor ($\emptyset_{material}$) of 0.88 to the measured laboratory strengths.

MODEL UNCERTAINTY

Introduction

The objective of the load magnification factor (in Equation 1) is to account for errors that come about when idealizing the situation for the purpose of determining the strength requirements. Depending on the analysis method and the extent of the simplifying assumptions, consequently the relevant load magnification factor would vary. Analysis of all methods is beyond the scope of this paper, however the most commonly adopted approaches typically include analytical and numerical approaches. The following sections consider these approaches.

Numerical Versus Analytical Solutions

Inherent in the derivation of the most commonly adopted analytical solutions for defining fill strength requirements (Mitchell *et al.*, 1982, Bloss 1992 and Baldwin & Grice, 1999) is the assumption of a rigid-perfectly plastic material behavior. While these solutions can provide a convenient strategy for determining the strength requirements, the behavior of cemented mine backfill is quite brittle and the implication of this rigid-perfectly plastic assumption should be properly understood and accounted for (via. $\emptyset_{model}$) when adopting these solutions.

The constitutive behavior of cemented mine backfill can vary considerably. Examples of this are shown in Figure 6, which presents the deviator stress (q) verse shear strain (e_q) from consolidated-drained (CD) triaxial testing of different cemented mine backfill materials. This figures shows that each of the materials yield at different strains and the degradation of strength with post-yield shear strain also differs.

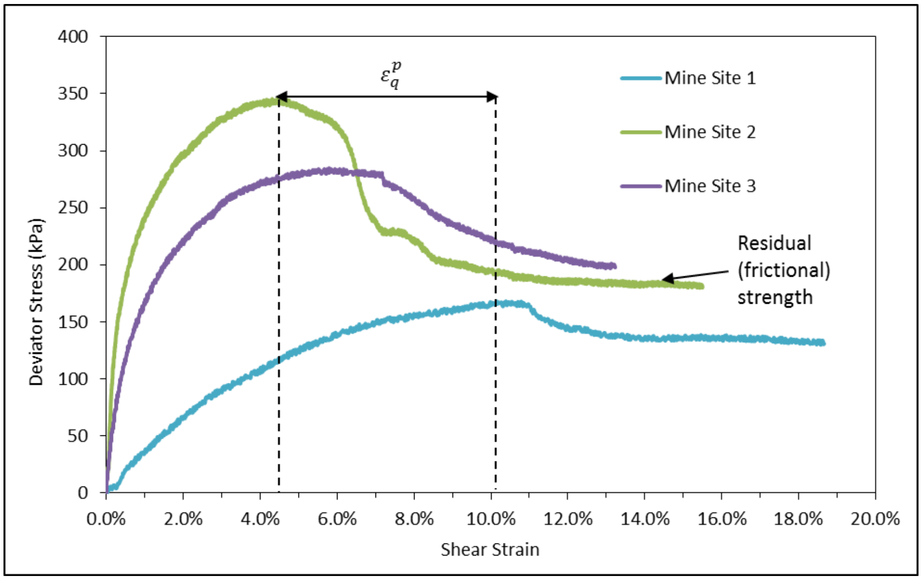
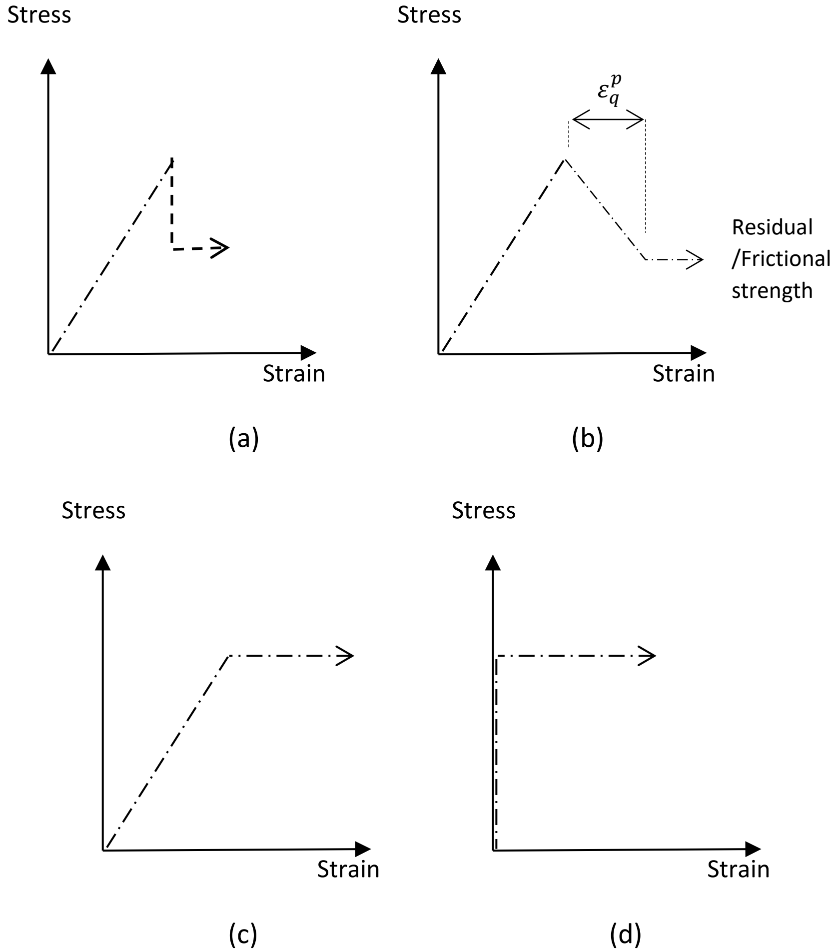


Figure 6. Stress-strain relationships from CD triaxial tests at various mine sites

All materials presented in Figure 6 demonstrate a tendency to strain soften quite considerably after reaching peak strength. This is inherent to all cemented materials, when sheared under relatively low confining stresses (as is the case in most cemented mine backfill exposures). This behavior contradicts the assumption of rigid perfect-plasticity.

As the assumption of rigid perfect-plasticity is inherent to all commonly adopted analytical solutions, it is considered important to understand the significance of this assumption on the results. To address this a numerical sensitivity study was undertaken. Analysis considered a range of post yield strain softening conditions. All models assume the same linear elasticity, however the post yield strain softening behavior was varied from perfectly brittle through to perfectly plastic. In addition, analysis also considered strain softening (i.e. complete bond strength degradation) to occur over post yield shear strain (ε_q^p) levels of 2%, 5% and 10%. Figure 7 illustrates the assumed material behavior for the different constitutive models on a deviator stress verses shear strain plot.



(a) (b)

(c) (d)

Figure 7. Shear stress verses shear strain for (a) Elastic Perfectly Brittle (b) Elastic strain-softening (c) Elastic Perfectly Plastic and (d) Rigid Perfectly Plastic constitutive behavior

Each model assumed a constant 50 m thick layer of constant strength and the fill strengths was progressively reduced until a failure mechanism formed. The minimum strength required for stability was determined and this is presented in Figure 8 for each of the constitutive models.

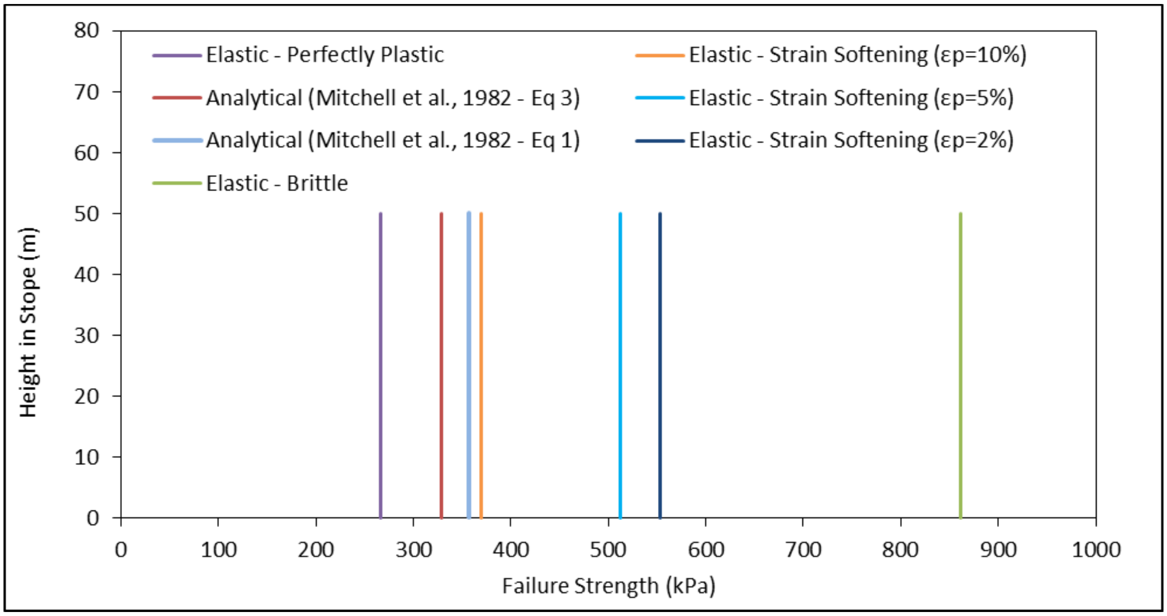


Figure 8. Strength required to maintain stability of a 20 m x 50 m tall vertical exposure with different constitutive models representing the fill material behavior

The results presented in Figure 8 show that the required fill strength is highly sensitive to the assumed constitutive model, with the strength requirements varying by a factor of 3 for the range of models considered. This result indicates that when using limit state numerical analysis it is critical to undertake the analysis using a constitutive model that properly represents the material behavior.

The results presented in Figure 8 also show that the strength requirement determined using the analytical solutions is similar to that determined from the numerical solutions where a ductile material response is assumed, which is intuitive considering the assumption of rigid perfect plasticity when deriving the analytical solution. This result suggests that one of the major contributing factors for the lower strength requirement, estimated using the analytical solution, is the rigid perfectly-plastic constitutive assumption inherent in the derivation of these solutions.

Typically cemented mine backfill material tends to strain-soften over a shear strain range (ε_q^p) of approximately 5%. The results presented in Figure 8 show that, relative to the strength requirement determined through rigorous numerical analysis, which properly incorporate the elastic strain-softening behavior of typical mine backfill, Mitchell et al.'s analytical solution (i.e. Equation 1 from Mitchell et al., 1982) underestimates the strength requirements by approximately 44%, while the more simplified analytical solution (i.e. Equation 3 from Mitchell et al., 1982) under estimates that strength requirements by approximately 56%.

CONCLUSION

The objective of this paper is to present the results of a numerical study that was undertaken for the purpose of providing a rational approach to the determination of safety factors appropriate for mine backfill design. This paper considered factors that should be applied to account for risk associated with both material and model uncertainty.

Reliability analysis was undertaken for the purpose of defining a rational framework for converting variable cemented mine backfill quality control strength test results to suitable mix relationships. The results indicate that, provided the fill strengths vary randomly according to a Gaussian distribution, when the coefficient of variation (CV) is less than 60%, the design fill strength can be taken as the value that 66% of quality control samples exceed. This can be implemented through the application of a strength reduction factor that reduces the mean strength by a factor equal to:

$$\emptyset_{material} = (1 - 0.413.CV)$$

To provide guidance for selecting a load magnification factor, which properly reflects model uncertainty associated with simplified analytical solutions a numerical sensitivity study was undertaken. This study showed that, relative to rigorous numerical analysis, the analytical solutions by Mitchell et al. (1982) tend to underestimate the strength requirements. Analysis indicates that the numerical and analytical solutions tend to converge as the ductility of the constitutive model adopted in the numerical analysis increases. This suggests that the assumption of rigid perfect plasticity, in deriving the analytical solutions, is the reason for the unconservative result. To address this an additional load magnification factor should be applied when determining fill strengths in accordance with more simplified analytical solutions. The results suggest that, for the conditions considered in this paper;

- when applying Mitchell et al.'s detailed analytical solution (Equation 1 from Mitchell et al., 1992) an additional load magnification factor of 1.44 should be applied.
- when applying Mitchell et al.'s simplified analytical solution (Equation 3 from Mitchell et al., 1992) an additional load magnification factor of 1.56 should be applied.

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